

SYLLABUS FOR AP CALCULUS (AB)

Department:	Mathematics
Course Title:	Calculus AB
Level and/or Grade:	AP; Grade 11/12
Prerequisite:	A- or better in Standard Precalculus or B- or better in Honors Precalculus or teacher recommendation

Instructional Programs:

Primary Textbook:

- Calculus: Graphical, Numerical, Algebraic; Finney/Thomas/Demana/Waits; Addison Wesley; 1995 (Copies of the 2007 version of this book are available for use to supplement the text in place.)

Supplementary Materials:

- AP Calculus AB/BC: Preparing for the Advanced Placement Examinations; Maxine Lifshitz; AMSCO Publication; 2004
- AP Calculus AB Course Requirements; College Board
- Multiple Choice & Free Response Questions in Preparation for the AP Calculus AB Examination; D & S Marketing, Inc.
- AP Central web resources

Mathematical Tools:

- TI-84 Graphing Calculators for modeling/representation
- MathCAD for concept development

Course Description:

This course is equivalent to a first-year college course in calculus. A theoretical foundation is laid through a treatment of functions, graphs, and limits; derivatives; and integrals. Emphasis is placed upon an understanding of the underlying principles of calculus rather than on memorizing formulas. Processes used include problem solving, reasoning, communication, representation, connections, and technology integration. Students electing this course are expected to take the Advance Placement Examination in May and, depending on the results, may be granted credit and/or appropriate placement by a participation college.

Learning Standards: *Through communication, representation, reasoning, making connections, and problem solving, students will be able to...*

I. Functions, Graphs, and Limits:

- Solve a variety of equations involving functions using graphical, numerical, analytical, or verbal methods; describe and analyze graphs using technology.
- Understand, calculate, and estimate limits from graphs or tables of data.
- Understand and describe asymptotic behaviors in terms of limits involving infinity; compare relative magnitudes of functions and their rates of change (e.g. contrasting exponential, polynomial, and logarithmic growth).

- Develop an intuitive understanding of continuity, and understand continuity in terms of limits.
- Develop a geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

II. Derivatives:

- Understand and develop the concepts of derivatives; define and interpret derivatives presented graphically, numerically, and analytically.
 - Interpret derivatives as instantaneous rates of change.
 - Define derivative as the limit of the difference quotient.
 - Understand the relationship between differentiability and continuity.
- Understand the derivative at a point: slope of curve at a point, tangent line to a curve at a point, rates of change.
- Understand derivatives as functions.
 - Solve equations involving derivatives; translate verbal descriptions into equations and vice versa.
 - Understand the concept of the Mean Value Theorem and its geometric consequences.
 - Discover the corresponding characteristics of graphs of f , f' , and f'' .
 - Understand the relationship between the increasing and decreasing behavior of f and the sign of f' .
 - Understand the relationship between the concavity of f and the sign of f'' .
 - Understand the concept of inflection points.
- Demonstrate an understanding of the relationship between the derivative and the definite integral.
- Apply derivatives.
 - Solve related rate problems including velocity, speed, and acceleration, derivatives of inverse functions, and geometric interpretations.
 - Analyze curves.
 - Solve optimization problems, including absolute and relative extreme.
 - Interpret differential equations geometrically; understand the relationship between slope fields and solution curves for differential equations.
- Understand derivatives as functions: power, exponential, logarithmic, and trigonometric.
- Understand the basic rules of computing derivatives (sums, products, quotients).

III. Integrals:

- Identify and understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change; solve authentic problems involving integrals.
- Understand basic properties of definite integrals (e.g. additivity, linearity).
- Apply and model integrals in physical, biological, or economic situations. Solve problems involving integrals: e.g. use integral rate of change to give accumulated change, find area of region, find volume of solid with cross sections, find average value of a function, find distance traveled by a particle along a line.
- Apply the Fundamental Theorem of Calculus to evaluate definite integrals and to represent an antiderivative.
- Understand techniques and applications of antidifferentiation.
- Apply antidifferentiation (e.g. motion along a line; modeling exponential growth).
- Understand the concept of the Riemann sum; use the Riemann sum and trapezoidal sum to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.
- Understand properties of definite integrals and their interpretations.

Course Alignment with High School Expectations for Student Learning:

Students will...

1. Analyze, interpret, evaluate and use logical reasoning to solve problems using a variety of resources and strategies.
 - Engage in a multi-representational approach to calculus with concepts, skills, and problems being expressed graphically, numerically, analytically, and verbally. Connections among these representations are made.
 - Use technology on a regular basis to enhance learning. Graphing calculators (TI-84) are used to plot graphs of functions, find zeros of functions, calculate derivative of a function, and calculate the value of a definite integral. MathCAD is used as an instructional strategy for developing lesson concepts.
 - Build new mathematical knowledge through problem solving. Solve AP Release Items – show enough work/steps so reasoning processes are evident; justify answers
 - Adapt and apply a variety of appropriate strategies to solve problems; reflect on the process of mathematical problem solving.
 - Monitor and reflect on the process of mathematical problems solving.
 - Recognize reasoning and proof as fundamental aspects of mathematics.
 - Make and investigate mathematical conjectures.
 - Solve problems that arise in mathematics and other contexts; use connections among mathematical ideas.
2. Communicate effectively to a variety of audiences.
 - Communicate mathematical thinking coherently and clearly to peers, teachers, and others - orally and through written work.
 - Use the language of mathematics to express ideas precisely.
3. Create works using a variety of communication forms.
 - Present arguments through writing; solve problems through projects, homework, tests, and quizzes; use technology; make oral presentations.
4. Develop skills and knowledge to reach personal and career goals.
 - Develop ‘habits of mind’: work beyond center of competence; gain attitude of persistence; seek feedback; develop confidence.
 - To receive college credit, prepare for, take, and pass the AP Calculus exam given in May.
5. Work cooperatively to achieve objectives.
 - Work in pairs, small groups, and part of the whole class to solve problems.
 - Analyze and evaluate the mathematical thinking and strategies of others.

Assessment:

Various assessment measures are used to monitor and measure students’ attainments of standards:

- Tests/quizzes
- Homework
- Projects
- Written Tasks & Oral Presentations
- Collaborative Tasks
- Participation/Effort

Scope & Sequence - Units of Study:

LIMITS & CONTINUITY – CHAPTER 2	TIMELINE
<p>Topics: <u>2.1 Limits:</u> Definition of Limit; Properties of Limits; One-sided and Two-sided Limits <u>2.2 Continuous Functions:</u> Continuity at a Point; Continuous Functions; Algebraic Combinations; Composites; Intermediate Value Theorem for Continuous Functions <u>2.3 The Sandwich Theorem</u> <u>2.4 Limits Involving Infinity:</u> Finite Limits as $x \rightarrow \infty$; End Behavior Models (Polynomial, Rational) & Asymptotes <u>2.5 Controlling Function Outputs - Target Values:</u> Aiming at the target; Controlling Outputs as $x = \infty$ <u>2.6 Defining Limits Formally w/Epsilons and Deltas:</u> Testing/Proving Limits; Finding Deltas for Given epsilons; Locally Straight Functions; 1-Sided vs. 2-Sided; Infinity</p> <p>Activities:</p> <ul style="list-style-type: none"> Integration of MathCAD computer-based interactive learning activities for 2.6. 	<p>Quarter 1</p>

DERIVATIVES – CHAPTER 3	TIMELINE
<p>Topics: <u>3.1 Slopes, Tangent Lines, & Derivatives:</u> Average Rates of Change; Slopes & Tangent Lines (Equations); Derivative of Function; Differentiable vs. Continuous Functions <u>3.2 Numerical Derivatives:</u> NDER Procedure; Graphs of Derivatives <u>3.3 Differentiation Rules:</u> Positive Integer Powers, Multiples, Sums, and Differences; Products and Quotients; Negative Integer Powers of x; Value Theorem for Derivatives; Second and Higher Order Derivatives <u>3.4 Velocity and Other Rates of Change:</u> Free Fall; Linear Animation; Velocity; Speed; Acceleration; Horizontal Motion; Other Rates of Change; Derivatives in Economics <u>3.5 Derivatives of Trigonometric Functions:</u> Derivative of the Sine Function; Derivative of the Cosine Function; Simple Harmonic Motion; Derivatives of Other Basic Trigonometric Functions <u>3.6 The Chain Rule:</u> The Chain Rule; Integer Powers of Differentiable Functions; “Outside-Inside” Rule; Derivative Formulas that Include the Chain Rule <u>3.7 Implicit Differentiation and Fractional Powers:</u> Graphing Curves; Implicit Differentiation; Lenses, Tangents, and Normal Lines; Derivatives of Higher Order; Fractional Powers of Differentiable Functions <u>3.8 Linear Approximations & Differentials:</u> Local Approximation; Linearizations (Linear Functions); Approximations; Estimating Change with Differentials; Absolute, Relative, & Percentage Change; Sensitivity; Approximation Error; Formulas for Differentials</p> <p>Activities:</p> <ul style="list-style-type: none"> Integration of MathCAD computer-based interactive learning activities for 3.7. 	<p>Quarter 1</p>

APPLICATIONS OF DERIVATIVES – CHAPTER 4	TIMELINE
<p>Topics:</p> <p><u>4.1 Maxima, Minima, & Mean Value Theorem:</u> Maxima and Minima – Local and Absolute; Mean Value Theorem; Rolle’s Theorem; Physical Interpretation; Increasing and Decreasing Functions; Other Consequences</p> <p><u>4.2 Predicting Hidden Behavior:</u> First Derivative; Second Derivative Test for Concavity; Points of Inflection; First Derivative Test for Local Extreme Values; Second Derivative Test for Local Extreme Values</p> <p><u>4.3 Polynomial Functions, Newton’s Method, & Optimization:</u> Polynomial Functions: Graphs of Cubic and Quartic polynomials; Newton’s Method; Differentials; Optimization</p> <p><u>4.4 Rational Functions & Economics Applications:</u> Graphs of rational functions; Optimization Examples from Mathematics, Business, Industry, and Economics</p> <p><u>4.5 Radical & Transcendental Functions:</u> Radical, Trigonometric, Exponential, and Logarithmic Functions</p> <p><u>4.6 Related Rates of Change:</u> Solving Related Rate Problems</p> <p><u>4.7 Antiderivatives, Initial Value Problems, & Mathematical Modeling:</u> Second and Third Corollaries of Mean Value Theorem; Finding Antiderivatives; Differential equations & Initial Value Problems; Solution Curves; Mathematical Modeling & Computer Simulations</p> <p>Activities:</p> <ul style="list-style-type: none"> • Integration of MathCAD computer-based interactive learning activities for 4.3 and 4.6. • Construct boxes that maximize volume given l and w of material used. • Sonobuoy Problem (Contraction Mapping Principle) • Largest Cone 	Quarter 2

INTEGRATION – CHAPTER 5	TIMELINE
<p>Topics:</p> <p><u>5.1 Calculus & Area:</u> Regions Bounded by Curves; Area under Graph of Nonnegative Continuous Function; Rectangular Approximation Method (RAM); Sigma Notation and Algebra Rules for Finite Sums; Computing Area</p> <p><u>5.2 Definite Integrals:</u> Riemann Sums; Terminology and Notation of Integration; Definite Integral and Area; NINT Procedure; Constant Functions; Integrals on a Calculator; Discontinuous Integrable Functions</p> <p><u>5.3 Definite Integrals & Antiderivatives:</u> Properties of Definite Integrals; Average Value of a Function; Mean Value Theorem for Definite Integrals; Connecting Differential and Integral Calculus</p> <p><u>5.4 The Fundamental Theorem of Calculus:</u> Fundamental Theorem, Part 1; Graphing the Function $\int_a^x f(t) dt$; Fundamental Theorem, Part 2; Area Connection; Analyzing Antiderivatives Graphically; Average Daily Inventory</p> <p><u>5.5 Indefinite Integrals:</u> Indefinite Integral of a function; Formulas and rules; Supporting Indefinite Integral Evaluation Graphically; Integrals of \sin^2x, \cos^2x, and $(\sin x)/x$; Initial value Problems</p>	Quarter 2

INTEGRATION – CHAPTER 5 (cont.)	TIMELINE
<p>5.6 <u>Integration by Substitution – Running the Chain Rule Backward</u>: Power Rule in Integral Form; Substitution Method of Integration; Substitution in Definite Integrals</p> <p>5.7 <u>Numerical Integration – The Trapezoidal Rule & Simpson’s Method</u>: Trapezoidal Rule; Simpson’s Rule; Error Analysis</p> <p>Activities:</p> <ul style="list-style-type: none"> Integration of MathCAD computer-based interactive learning activities for 5.2 and 5.7. 	Quarter 2

APPLICATIONS OF DEFINITE INTEGRALS – CHAPTER 6	TIMELINE
<p>Topics:</p> <p>6.1 <u>Areas Between Curves</u>: Area Between Curves –Basic formula; Area Enclosed by Intersecting Curves; Boundaries with Changing Formulas; Integrating with Respect to y; Combining Integrals with Geometry Formulas</p> <p>6.2 <u>Volumes of Solids of Revolution-Disks and Washers</u>: The Disk Method; Revolving About the y-axis; Solids of Revolution</p> <p>6.3 <u>Cylindrical Shells – An Alternative to Washers</u>: Basic Shell Formula; Shells About the x-axis; Shifts; Overlapping Regions</p> <p>6.4 <u>Lengths of Curves in the Plane</u>: Basic Formula; Discontinuities in dy/dx; Short Differential Formula; Curves with Infinite Length</p> <p>6.5 <u>Areas of Surfaces in Revolution</u>: Basic Formula; Revolution About the y-axis; Short Differential Formula</p> <p>6.6 <u>Work</u>: Constant-Force Formula; Hooke’s Law</p> <p>Activities:</p> <ul style="list-style-type: none"> Integration of MathCAD computer-based interactive learning activities for 6.1, 6.2, and 6.3. 	Quarter 3

THE CALCULUS OF TRANSCENDENTAL FUNCTIONS – CHAPTER 7	TIMELINE
<p>Topics:</p> <p>7.1 <u>The Natural Logarithm Function</u>: Natural Logarithm function; derivative of $y = \ln x$; Logarithmic Differentiation; Integrals of $\int \frac{1}{u} du$, $\tan x$ and $\cot x$</p> <p>7.2 <u>The Exponential Function</u>: Exponential functions; Rules of Exponents; Derivatives of Inverses of Differentiable Functions; Law of Exponential Change</p> <p>7.3 <u>Other Exponential & Logarithmic Functions</u>: Function and Graph of a^x; Power rule for Differentiation – Final Form; Exponential & Logarithmic Functions</p> <p>7.4 <u>The Law of Exponential Change Revisited</u>: Law of Exponential Change; Radioactive Decay; Resistance to moving Object; Heat Transfer</p> <p>7.5 <u>Indeterminate Forms & L’Hopital’s Rule</u>: Indeterminate Forms; L’Hopital’s Rule</p> <p>7.6 <u>The Rates at Which Functions Grow</u>: Relative Rates of Growth; Sequential vs. Binary Search</p>	Quarter 3

THE CALCULUS OF TRANSCENDENTAL FUNCTIONS – CHAPTER 7 (cont.)	TIMELINE
<p>7.7 <u>The Inverse Trigonometric Functions</u>: Inverse Trigonometric Functions (Inverse of $\sec x$, $\csc x$, and $\cot x$); Arc Sine, Arc Cosine, and Arc Tangent; Right Triangle Interpretations</p> <p>7.8 <u>Derivatives of Inverse Trigonometric Functions – Related Integrals</u>: Derivatives of Inverse Trigonometric Functions</p> <p>7.9 <u>Hyperbolic Functions</u>: Hyperbolic Functions; Derivatives and Integrals; Inverses of Hyperbolic Functions</p> <p>Activities:</p> <ul style="list-style-type: none"> • Growth & Decay Problems (e.g. spread of disease) • Compound Interest and Consumer Price Index Problems • Radioactive Decay Problems – half-life • Integration of MathCAD computer-based interactive learning activities for 7.9. 	<p>Quarter 3</p>

TECHNIQUES OF INTEGRATION – CHAPTER 8	TIMELINE
<p>Topics:</p> <p>8.1 <u>Formulas for Elementary Integrals</u>: Algebraic Procedures & Trigonometric Identities; Mercator's World Map</p> <p>8.2 <u>Integration by Parts</u>: The Formula; Tabular Integration</p> <p>8.3 <u>Integrals Involving Trigonometric Functions</u>: Products of Sines and Cosines; Integrals of Powers of $\tan x$ and $\sec x$; Definite Integrals of Odd and Even Functions</p> <p>8.4 <u>Trigonometric Substitutions</u>: Combining Squares; Formulas; Completing Squares</p> <p>8.5 <u>Rational Functions & Partial Functions</u>: Partial Fractions</p> <p>8.6 <u>Improper Integrals</u>: Convergence & Divergence; Domination Test; Limit Comparison Test; Two infinite Limits of Integration</p> <p>Activities:</p> <ul style="list-style-type: none"> • Integration of MathCAD computer-based interactive learning activities for 8.2. 	<p>2nd half of Quarter 3 and 1st half of Quarter 4</p>

INFINITE SERIES – CHAPTER 9	TIMELINE
<p>Topics:</p> <p>9.1 <u>Limits of Sequences & Series</u>: Defining a Sequence; Notations; Graphing a Sequence; Limit of a Sequence; Convergence & Divergence; Useful Theorems</p> <p>9.2 <u>Infinite Series</u>: Sequence of Partial Sums; Geometric Series; Sums of Other Convergent Series; Test for Divergence; Useful Theorems; Perpetuities</p> <p>9.3 <u>Series without Negative Terms – Comparison & Integral Tests</u>: Nondecreasing Sequences; Comparison Test for Convergence; Integral test; Limit Comparison Test</p> <p>9.4 <u>Series with Nonnegative Terms – Ratio & Roots Tests</u>: Comparing Nonnegative Series; Ratio Test; nth Root Test</p> <p>9.5 <u>Alternating Series & Absolute Convergence</u>: Alternating Series Theorem; Absolute Convergence; Power Series</p>	<p>Quarter 4</p>

INFINITE SERIES – CHAPTER 9	TIMELINE
<p><u>9.6 Power Series</u>: Power Series and Convergence; Radius and Interval of Convergence; Arctangents; Term-by-term Integration of power Series; Multiplication of Power Series</p> <p><u>9.7 Taylor Series & Maclaurin Series</u>: Taylor Polynomials; Maclaurin and Taylor Series; Taylor Series with Remainder; Remainder estimation; Combining Taylor Series</p> <p><u>9.8 Further Calculations with Taylor Series</u>: Binomial Series; Choosing Centers for Taylor Series; Evaluating Nonelementary Integrals;</p>	<p>Quarter 4</p>